Lie Dragging of Advected Invariants in ideal Magnetohydrodynamics and Gas Dynamics

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Abstract: We discuss methods to obtain physical quantities that are advected with the flow in ideal gas dynamics (fluid mechanics) and magnetohydrodynamics (MHD). Thus, for example, the entropy S is a scalar (function or 0-form) that is advected with the flow, meaning the Lagrangian time derivative of S moving with the flow is zero. Similarly the magnetic flux (which is a two form) is advected with the flow, and corresponds to Faraday's equation in ideal MHD. Thus, the entropy and the magnetic flux are said to be Lie dragged with the flow. A motivation for our analysis is to better understand magnetic helicity and other physical quantities that describe the magnetic field topology, and their relationships to conservation laws. In ideal fluid dynamics, the potential vorticity is a scalar invariant that is Lie dragged with the flow. In Lagrangian fluid mechanics, one can show that the potential vorticity or Ertel invariant is due to fluid relabeling symmetries and Noether's second theorem (which we do not discuss in the talk). A higher order invariant related to the Ertel invariant by Hollman is discussed. There appear to be an infinite number of invariants that are Lie dragged with the flow, some of which are important in topological fluid dynamics and MHD. We discuss how these invariants can be derived using the Calculus of exterior differential forms orginally developed by Elie Cartan.